

PRE COPY

(4)

ARL-STRUC-R-438

AR-005-592



DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

AD-A213 998

Aircraft Structures Report 438

A PENALTY ELEMENT FORMULATION FOR
CALCULATING BULK STRESS

by

W. Waldman, T.E. Tay and R. Jones

DTIC
ELECTE
NOV 02 1989
S E D
(Handwritten mark)

Approved for public release.

(C) COMMONWEALTH OF AUSTRALIA 1989
FEBRUARY 1989

89 11 01 055

THE GOVERNMENT OF AUSTRALIA
TECHNICAL INFORMATION SERVICE
IS AUTHORIZED TO
REBODGE AND SELL THIS REPORT

This work is copyright. Apart from any fair dealing for the purpose of study, research, criticism or review, as permitted under the Copyright Act, no part may be reproduced by any process without written permission. Copyright is the responsibility of the Director Publishing and Marketing, AGPS. Inquiries should be directed to the Manager, AGPS Press, Australian Government Publishing Service, GPO Box 84, CANBERRA ACT 2601.

AR-005-592

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORY

Aircraft Structures Report 438

**A PENALTY ELEMENT FORMULATION FOR
CALCULATING BULK STRESS**

by

W. Waldman, T.E. Tay and R. Jones

SUMMARY

This report describes the formulation of a three-dimensional penalty element that uses the bulk stress at each node as a degree of freedom, in addition to the usual u, v, and w displacements. The penalty function method is used to define a modified energy functional that is constrained by the equation for bulk stress, and a solution is obtained by specifying stationarity of this modified functional. Edge-cracked and centre-cracked panels are modelled in order to investigate the practical implementation of the penalty element formulation and to compare the results with those obtained using standard methods.



(C) COMMONWEALTH OF AUSTRALIA 1989

POSTAL ADDRESS: Director, Aeronautical Research Laboratory,
P.O. Box 4331, Melbourne, Victoria, 3001, Australia

Page 1

CONTENTS

1.0 INTRODUCTION	1
2.0 MATHEMATICAL FORMULATION	2
3.0 VALIDATION OF PENALTY ELEMENT FORMULATION	6
3.1 Edge-cracked and centre-cracked panels	7
3.2 Bulk stress calculations	7
3.3 Stress intensity factor calculations	8
4.0 CONCLUSION	9
REFERENCES	11
TABLES	13
FIGURES	
DISTRIBUTION	
DOCUMENT CONTROL DATA	

Accession For	
NTIS GRATI	
DTIC TAB	
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	



1.0 INTRODUCTION

In recent years the thermoelastic effect has been greatly exploited as a non-contact method of experimental stress analysis. A device known as SPATE (Stress Pattern Analysis by measurement of Thermal Emission) has been used extensively at the Aeronautical Research Laboratory to analyze many different components to gain data on the surface stress distributions. The SPATE apparatus accomplishes its analysis by measuring the thermal emission of the surface of the component which is being sinusoidally loaded about some mean load. The thermal emission is proportional to the surface temperature and, because the thermal fluctuations are proportional to the changes in bulk stress ($\sigma_{zz} + \sigma_{yy} + \sigma_{xx}$), a strain gauge can be used to calibrate the output from SPATE in order to determine the actual bulk stress.

Although the use of SPATE for the analysis of components is a powerful and versatile technique, the tensorial nature of stress is ignored since temperature and bulk stress are scalar quantities. This places a significant limitation on the ability of SPATE data to provide a quantitative measure of the stress field, except in circumstances where it is known that the stress field is primarily uni-directional. Another disadvantage is that no direct information concerning the nature of the internal stress field of a three-dimensional structure can be obtained from the simple surface analysis performed by SPATE.

It is well known that the stress field existing in a component must satisfy equilibrium and any imposed boundary conditions. For two-dimensional problems, Ryall and Wong [1] have shown that when additional stress-related information is available, such as experimentally obtained bulk stress data from a SPATE analysis, it is no longer necessary to have a complete knowledge of the boundary conditions governing the problem in order to obtain a useful solution. Their method involved the use of an assumed stress potential Φ , with unknown coefficients, from which the bulk stress could be calculated analytically. The values of the unknown coefficients were determined by formulating the problem using a best-fit criterion where the difference between the analytical expression for the bulk stress and the experimentally observed bulk stresses was minimized in the least squares sense.

This report describes the formulation of an alternative finite element approach using a three-dimensional penalty element that uses the bulk stress at each node as a degree of freedom, in addition to the usual u , v , and w displacements. The penalty function method, as described in [2], will be used to define a modified energy functional that is constrained by the equation for bulk stress, a solution being obtained by specifying stationarity of this functional.

This approach leads to an interesting possible application where a finite element model of a component is created, and the bulk stress, as obtained by an experimental technique such as SPATE, is prescribed at various points. The resulting system can then be solved, making use of any other available data concerning the applied loads and boundary conditions, thus determining the full three-dimensional stress field in the component. In problems where ill-conditioning is

encountered, such as adhesively bonded repairs, it may also be possible to use the measured bulk stress field as a means of improving the conditioning, thus driving the solution to a more reliable answer.

2.0 MATHEMATICAL FORMULATION

In the finite element method the displacements \mathbf{u} at any point within an element may be written as

$$\mathbf{u} = \mathbf{N}\mathbf{a}, \quad (2.1)$$

where the components of \mathbf{N} are prescribed functions of position and \mathbf{a} represents the vector of nodal displacements for a particular element. The vector \mathbf{a} may be written in terms of the n nodal points as

$$\mathbf{a} = \begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_n \end{Bmatrix}, \quad (2.2)$$

and if each \mathbf{a}_i is allowed horizontal, lateral, and vertical displacement degrees of freedom then we can write

$$\mathbf{a}_i = \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}. \quad (2.3)$$

With the displacements known at all nodal points, then by using the linear theory of elasticity the strains can be written as

$$\boldsymbol{\epsilon} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{Bmatrix} = \mathbf{B}\mathbf{u}, \quad (2.4)$$

where

$$\mathbf{B} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{pmatrix} \quad (2.5)$$

and the relationship between stress and strain is of the form

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{pmatrix} = \mathbf{D}\boldsymbol{\epsilon}, \quad (2.6)$$

where \mathbf{D} is an elasticity matrix containing the appropriate material properties.

We will now introduce the definitions of the bulk stress σ_b , the bulk strain e , and the bulk modulus of the material κ :

$$\sigma_b = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \quad (2.7)$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z \quad (2.8)$$

$$\kappa = \frac{E}{1 - 2\nu} \quad (2.9)$$

where σ_{xx} σ_{yy} σ_{zz} are the principal stresses, and ϵ_x ϵ_y ϵ_z are the principal strains, E is Young's modulus, and ν is Poisson's ratio.

The equation relating the bulk stress σ_b to the bulk strain e is

$$\sigma_b = \kappa e \quad (2.10)$$

$$= \kappa(\epsilon_x + \epsilon_y + \epsilon_z) \quad (2.11)$$

$$= \boldsymbol{\kappa}^T \boldsymbol{\epsilon}. \quad (2.12)$$

and the bulk modulus vector $\boldsymbol{\kappa}$ is defined as

$$\kappa = \begin{pmatrix} \kappa \\ \kappa \\ \kappa \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (2.13)$$

Equation (2.12) may be written in the form

$$\sigma_b - \kappa^T \epsilon = 0 \quad (2.14)$$

and we note that the bulk stress σ_b is a scalar quantity.

We now proceed to define an energy functional Π which, in the absence of body forces, is a combination of the standard energy functional Π_a due to elastic deformations, and a penalty function term Π_σ which involves the bulk stress σ_b , such that

$$\Pi(\epsilon, \sigma_b) = \Pi_a + \Pi_\sigma \quad (2.15)$$

where

$$\Pi_a = \frac{1}{2} \int \epsilon^T D\epsilon dV - \int u^T T dS \quad (2.16)$$

$$\Pi_\sigma = \frac{P}{2} \int (\sigma_b - \kappa^T \epsilon)^2 dV. \quad (2.17)$$

and T is the surface traction vector $\{T_x, T_y, T_z\}^T$.

Now, by substituting for u , the strains ϵ can be written in terms of the nodal displacements as follows

$$\epsilon = BN\alpha. \quad (2.18)$$

Hence, the standard energy functional Π_a may be expressed as

$$\Pi_a = \frac{1}{2} \alpha^T K \alpha - \alpha^T f_T \quad (2.19)$$

where the stiffness matrix K and the force vector f_T are given by

$$K = \int N^T B^T D B N dV \quad (2.20)$$

$$f_T = \int N^T T dS \quad (2.21)$$

and the penalty functional Π_σ becomes

$$\Pi_\sigma = \frac{P}{2} \int (\sigma_b^2 - 2\sigma_b \kappa^T \epsilon + \kappa^T \epsilon \kappa^T \epsilon) dV \quad (2.22)$$

$$= \frac{P}{2} \int (\sigma_b^2 - 2\sigma_b \kappa^T \mathbf{B} \mathbf{N} \mathbf{a} + \mathbf{a}^T \mathbf{N}^T \mathbf{B}^T \kappa \kappa^T \mathbf{B} \mathbf{N} \mathbf{a}) dV \quad (2.23)$$

$$= \frac{P}{2} \int \sigma_b^2 dV - P \int \sigma_b \kappa^T \mathbf{B} \mathbf{N} dV \mathbf{a} + \frac{P}{2} \mathbf{a}^T \mathbf{K}_{aa} \mathbf{a} \quad (2.24)$$

where

$$\mathbf{K}_{aa} = \int \mathbf{N}^T \mathbf{B}^T \kappa \kappa^T \mathbf{B} \mathbf{N} dV \quad (2.25)$$

$$= \kappa^2 \int \mathbf{N}^T \mathbf{J} \mathbf{N} dV \quad (2.26)$$

and

$$\mathbf{J} = \begin{pmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x \partial z} \\ \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial y \partial z} \\ \frac{\partial^2}{\partial z \partial x} & \frac{\partial^2}{\partial z \partial y} & \frac{\partial^2}{\partial z^2} \end{pmatrix} \quad (2.27)$$

and P is a penalty parameter whose nominal value is assigned such that the order of magnitude of the two functionals Π_a and Π_σ is approximately the same. In our particular case, it will be shown later that the penalty parameter can be specified to be equal to either $1/E$ or $1/\kappa$ without significant change in the solution.

Let us now define σ_b in terms of a shape function \mathbf{M} and the values of the bulk stress at the nodal points, the latter denoted by the bulk stress vector \mathbf{b} , such that

$$\sigma_b = \mathbf{M}^T \mathbf{b}, \quad (2.28)$$

and by noting that $\mathbf{M}^T \mathbf{b} = \mathbf{b}^T \mathbf{M}$ we obtain a new expression for the penalty functional

$$\Pi_\sigma = \frac{P}{2} \int \mathbf{b}^T \mathbf{M} \mathbf{M}^T \mathbf{b} dV - P \int \mathbf{b}^T \mathbf{M} \kappa^T \mathbf{B} \mathbf{N} dV \mathbf{a} + \frac{P}{2} \mathbf{a}^T \mathbf{K}_{aa} \mathbf{a} \quad (2.29)$$

$$= \frac{P}{2} \mathbf{b}^T \mathbf{K}_{bb} \mathbf{b} - P \mathbf{b}^T \mathbf{K}_{ba} \mathbf{a} + \frac{P}{2} \mathbf{a}^T \mathbf{K}_{aa} \mathbf{a} \quad (2.30)$$

where

$$\mathbf{K}_{bb} = \int \mathbf{M} \mathbf{M}^T dV \quad (2.31)$$

$$\mathbf{K}_{ba} = \int \mathbf{M} \kappa^T \mathbf{B} \mathbf{N} dV. \quad (2.32)$$

Hence the complete expression for the energy functional Π is

$$\Pi(\mathbf{a}, \mathbf{b}) = \frac{1}{2} \mathbf{a}^T \mathbf{K} \mathbf{a} - \mathbf{a}^T \mathbf{f}_T + \frac{P}{2} \mathbf{b}^T \mathbf{K}_{bb} \mathbf{b} - P \mathbf{b}^T \mathbf{K}_{ba} \mathbf{a} + \frac{P}{2} \mathbf{a}^T \mathbf{K}_{aa} \mathbf{a} \quad (2.33)$$

where Π is now a function of the displacement degrees of freedom \mathbf{a} and the bulk stress degrees of freedom \mathbf{b} specified at each node.

In order to obtain a solution, we apply the principle of stationarity to the energy functional Π , namely

$$\frac{\partial \Pi}{\partial \mathbf{a}} = \mathbf{0} \quad (2.34)$$

$$\frac{\partial \Pi}{\partial \mathbf{b}} = \mathbf{0} \quad (2.35)$$

and we obtain the following expressions

$$\frac{\partial \Pi}{\partial \mathbf{a}} = \mathbf{K} \mathbf{a} - \mathbf{f}_T + P \mathbf{K}_{aa} \mathbf{a} - P \mathbf{K}_{ba} \mathbf{b} \quad (2.36)$$

$$\frac{\partial \Pi}{\partial \mathbf{b}} = -P \mathbf{K}_{ba} \mathbf{a} + P \mathbf{K}_{bb} \mathbf{b}. \quad (2.37)$$

Hence, the stationarity condition leads to a set of linear equations, which can be expressed in matrix form as

$$\left(\begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + P \begin{bmatrix} \mathbf{K}_{aa} & -\mathbf{K}_{ba} \\ -\mathbf{K}_{ba} & \mathbf{K}_{bb} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_T \\ \mathbf{0} \end{Bmatrix}. \quad (2.38)$$

We note that the term involving \mathbf{K} is just the standard finite element formulation of the stiffness matrix, while the matrix term multiplied by the penalty parameter P originates from the penalty element formulation for the bulk stress degrees of freedom.

3.0 VALIDATION OF PENALTY ELEMENT FORMULATION

In order to check the theoretical formulation of the penalty element approach, as well as its practical implementation into a finite element package, two demonstration problems will be analyzed. These consist of three-dimensional finite element models of an edge-cracked panel and a centre-cracked panel, from which it is possible to determine bulk stresses and displacements at various nodes in the finite element mesh. Figures 1 and 2 show the general dimensions of the edge-cracked and centre-cracked panels that were studied.

The test problems were analyzed using the PAFEC finite element package running on the ELXSI 6400 computer at ARL. The penalty element formulation was incorporated into the PAFEC program by re-writing the code that deals with 20-noded orthotropic brick elements in order to incorporate the penalty element.

The penalty element was assigned an element-type number of 37150, and this element type overwrites the PAFEC isotropic element of the same name. The orthotropic element type was chosen for this modification because the required changes were easier to code than for an isotropic element.

3.1 Edge-cracked and centre-cracked panels

Figure 3 shows the finite element mesh used for modelling the edge-cracked panel, and it details the major dimensions, including those of the simulated crack. Due to symmetry only 1/4 of the structure is shown, and the full model represented by this mesh can be obtained by reflection in the $X-Z$ and $X-Y$ planes. Therefore, the nodes along the x -axis correspond to interior points in the plate. The loading applied to the finite element model consists of a uniform prescribed v -displacement of 0.01 mm along its top surface (see Figures 1 and 2), and it should be noted that restraints are applied on that face to prevent any strains in the x -direction.

The centre-cracked panel was modelled by applying additional restraints to the finite element model of the edge-cracked panel, making use of the existing mesh. Hence, with the appropriate restraints, the finite element mesh shown in Figure 3 also corresponds to 1/8 of the full centre-cracked panel, and the full model can be obtained by reflecting the mesh in the $X-Y$, $X-Z$ and $Y-Z$ planes. The loading for the centre-cracked panel was identical to that used for the edge-cracked panel.

The material chosen for the two different types of panel corresponds to an epoxy adhesive, whose material properties are presented in Table 1. Although these properties are input as for an orthotropic material, they are chosen so as to make the material behave in an isotropic manner. The data are given for two different values of Poisson's ratio, where $\nu = 0.30$ corresponds to the values used for the centre-cracked panel, and $\nu = 0.35$ corresponds to the edge-cracked panel. Initially some of the finite element calculations were carried out using $\nu = 0.35$, but in order to match the Poisson's ratio used for obtaining the stress intensity factor for a centre-cracked panel [3], it was also necessary to use a Poisson's ratio of $\nu = 0.30$.

Figure 4 shows a typical deformed finite element mesh that was obtained for the edge-cracked panel when it was loaded by a uniform displacement. The deformation of the mesh in the vicinity of the crack tip can be seen clearly.

3.2 Bulk stress calculations

The edge-cracked and centre-cracked panels were analyzed using both the standard and penalty element formulations. For the standard element case, bulk stresses were calculated at a number of nodes ahead of the crack tip by simply summing the principal stresses according to Equation (2.7). Since the nodes chosen for this comparison were placed at element boundaries, the bulk stresses were calculated as the average of the results obtained for adjacent elements.

Tables 2 and 3 present the bulk stresses calculated for the edge-cracked and centre-cracked panels. When using the penalty element formulation, two different values of the penalty parameter were tried, the first corresponding to $P = 1/E$ and the second to $P = 1/\kappa$. The data presented in these two tables indicates that there is very good agreement between the standard and the penalty element formulations. In areas where there is a rapid stress variation, the answers generally agree to within 7%, while the average difference is about 4%. Note that the results obtained for the two different values of the penalty parameter are also in very good agreement, indicating that the solution is not very sensitive to changes in the value of the penalty parameter when it is of the order of magnitude $O(1/E)$. Also note that there is a stress singularity at node 12, since this is right at the crack tip, and consequently the finite element results are inaccurate at that location; the results for node 12 were included only to enable a qualitative comparison to be made between the standard and penalty element formulations.

3.3 Stress intensity factor calculations

In order to give the required \sqrt{r} displacement behaviour for near crack tip points in the vicinity of the crack front, special elements were used that had their midside nodes shifted to the quarter point position. The value of the stress intensity factor K_I near the crack front can be determined from the opening displacement of the crack near the tip using the plane strain equation

$$K_I = \frac{\delta E}{4(1-\nu^2)} \sqrt{\frac{2\pi}{r(1-\frac{r}{2l})}} \quad (3.1)$$

where

δ is the displacement in the y -direction of a point lying in the $X-Z$ plane near the crack front

r is the perpendicular distance of the point from the crack front

l is the crack length for an edge-cracked panel, and is the crack half-length for a centre-cracked panel

E is Young's modulus

ν is Poisson's ratio.

Table 4 presents values of the stress intensity factor K_I as obtained using both the standard and penalty element formulations, and compares them to the values of K_I given by Rooke & Cartwright [3] for both an edge-cracked and centre-cracked panel. The calculations were performed for an applied v -displacement of 0.01 mm, and it is seen that the two different finite element formulations give results that are in good agreement with those obtained from [3]. Note that the results obtained using standard three-dimensional elements are closer to those given in [3], but in any case the maximum difference is only about 4%, which is not very significant

when the accuracy of the results from [3] are quoted as being within an error bound of a similar magnitude. However, the difference in values of K_I calculated using standard and penalty elements is only about 1.5%, which is very small.

Tables 5 and 6 present values of crack opening displacement δ , stress intensity factor K_I , and reaction force F_y obtained for the edge-cracked and centre-cracked panels. In order to study the sensitivity of the penalty element solution to different values of the penalty parameter P , a wide range of values were used, with $P = 100$ being the largest value that could be used without numerical problems. The maximum variation in crack opening displacement and stress intensity factor experienced for the edge-cracked panel was 1.5%, while for the centre-cracked panel it was 2.0%.

4.0 CONCLUSION

By using a penalty element formulation, the standard finite element method has been modified to allow the calculation of bulk stresses as an additional degree of freedom in the finite element model of a structure. The bulk stress penalty elements have been incorporated into the PAFEC finite element package.

In order to check the practical implementation of the penalty element formulation, edge-cracked and centre-cracked panels were modelled, and the solutions obtained were compared with those calculated using the standard finite element method. Also, both the standard and penalty element formulations have been used to obtain crack opening displacements, from which the values of K_I were calculated and compared with those from [3].

The results of these comparisons indicate that bulk stresses can be accurately predicted using the penalty element method, and that there is no significant effect on the quality of the results obtained for the other degrees of freedom. It has also been found that the present application of the penalty element formulation is very insensitive to the choice of penalty parameter P , and a suitable value for P can be defined by using either Young's modulus or the bulk modulus, the appropriate values being given by $P = 1/E$ or $P = 1/\kappa$.

It is envisaged that this methodology will subsequently be used, in conjunction with experimentally obtained values for the bulk stress, to develop a hybrid experimental/numerical procedure for determining individual stress components based on the work presented in [1].

REFERENCES

- [1] T.G. Ryall and A.K. Wong, Determining Stress Components From Thermoelastic Data - A Theoretical Study, Aircraft Structures Technical Memorandum 291, Aeronautical Research Laboratory, Defence Science and Technology Organisation, Australia, June 1988. (Also published in Mechanics of Materials, pp. 205-214, 1988.)
- [2] O.C. Zienkiewicz, The Finite Element Method, Third Edition, McGraw-Hill Book Company (UK) Limited, 1977.
- [3] D.P. Rooke and D.J. Cartwright, Compendium of Stress Intensity Factors, Her Majesty's Stationery Office, London, 1976.

TABLE 1
 Mechanical Properties of Adhesive For
 Two Values of Poisson's Ratio

MECHANICAL PROPERTY (MPa ⁻¹)	CENTRE-CRACKED PANEL $\nu = 0.30$	EDGE-CRACKED PANEL $\nu = 0.35$
S_{xx}	5.2910×10^{-4}	5.2910×10^{-4}
S_{yy}	5.2910×10^{-4}	5.2910×10^{-4}
S_{zz}	5.2910×10^{-4}	5.2910×10^{-4}
S_{xy}	-1.5873×10^{-4}	-1.8519×10^{-4}
S_{yz}	-1.5873×10^{-4}	-1.8519×10^{-4}
S_{zx}	-1.5873×10^{-4}	-1.8519×10^{-4}
SH_{xy}	1.3757×10^{-3}	1.4286×10^{-3}
SH_{yz}	1.3757×10^{-3}	1.4286×10^{-3}
SH_{zx}	1.3757×10^{-3}	1.4286×10^{-3}

TABLE 2

Bulk Stress at Different Nodes Calculated Using
Standard and Penalty Elements with Different
Values of Penalty Parameter P

EDGE-CRACKED PANEL					
NODE NO.	DISTANCE FROM CRACK FACE (mm)	BULK STRESS (MPa)			
		Standard Element	Penalty Element	$P = 1/E$	$P = 1/\kappa$
12	0.0000	26.31	27.28	28.09	
13	0.1667	12.29	11.47	11.48	
14	0.5000	5.19	5.11	5.08	
15	1.6667	2.65	2.75	2.74	
16	5.0000	2.02	2.03	2.01	

TABLE 3

Bulk Stress at Different Nodes Calculated Using
Standard and Penalty Elements with Different
Values of Penalty Parameter P

CENTRE-CRACKED PANEL					
NODE NO.	DISTANCE FROM CRACK FACE (mm)	BULK STRESS (MPa)			
		Standard Element	Penalty Element	$P = 1/E$	$P = 1/\kappa$
12	0.0000	21.70	22.72	23.09	
13	0.1667	9.98	9.48	9.43	
14	0.5000	4.65	4.38	4.37	
15	1.6667	2.48	2.57	2.55	
16	5.0000	2.00	2.02	2.01	

TABLE 4
 Values of Stress Intensity Factor K_I as Obtained Using Different Methods

SOLUTION METHOD	STRESS INTENSITY FACTOR K_I (MPa \sqrt{m})	
	EDGE-CRACKED PANEL	CENTRE-CRACKED PANEL
Rooke & Cartwright [3]	0.155	0.140
Standard Elements	0.151	0.137
Penalty Elements ($P = 1/E$)	0.149	0.135

TABLE 5

Finite Element Results For Crack Opening, Stress Intensity Factor, and Reaction Force Obtained With Different Values of Penalty Parameter P

EDGE-CRACKED PANEL			
PENALTY PARAMETER P	CRACK OPENING δ AT NODE 11 (m)	STRESS INTENSITY FACTOR K_I (MPa \sqrt{m})	REACTION F_y AT NODE 3 (N)
$0.000 \times 10^{+0}*$	1.4062×10^{-6}	0.1508	18.299
1.000×10^{-6}	1.4061×10^{-6}	0.1508	18.299
1.000×10^{-5}	1.4051×10^{-6}	0.1507	18.300
1.000×10^{-4}	1.3984×10^{-6}	0.1500	18.304
$1.587 \times 10^{-4}\dagger$	1.3956×10^{-6}	0.1497	18.306
$5.291 \times 10^{-4}\ddagger$	1.3874×10^{-6}	0.1488	18.314
1.000×10^{-2}	1.3778×10^{-6}	0.1478	18.338
$1.000 \times 10^{+0}$	1.3858×10^{-6}	0.1486	18.362
$1.000 \times 10^{+1}$	1.3909×10^{-6}	0.1492	18.413
$1.000 \times 10^{+2}$	1.4280×10^{-6}	0.1532	18.827

* standard finite element formulation

† corresponds to $P = 1/\kappa$

‡ corresponds to $P = 1/E$

TABLE 6

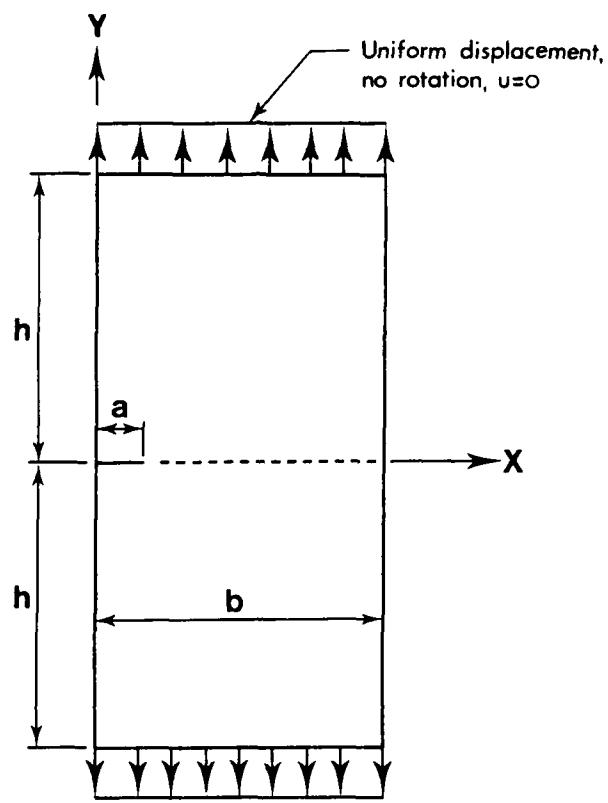
Finite Element Results For Crack Opening, Stress Intensity Factor, and Reaction Force Obtained With Different Values of Penalty Parameter P

CENTRE-CRACKED PANEL			
PENALTY PARAMETER P	CRACK OPENING δ AT NODE 11 (m)	STRESS INTENSITY FACTOR K_I (MPa \sqrt{m})	REACTION F_y AT NODE 3 (N)
$0.000 \times 10^{+0}*$	1.3211×10^{-6}	0.1367	18.549
1.000×10^{-6}	1.3211×10^{-6}	0.1367	18.549
1.000×10^{-5}	1.3205×10^{-6}	0.1366	18.549
1.000×10^{-4}	1.3160×10^{-6}	0.1361	18.552
$2.116 \times 10^{-4}\dagger$	1.3124×10^{-6}	0.1358	18.554
$5.291 \times 10^{-4}\ddagger$	1.3067×10^{-6}	0.1352	18.558
1.000×10^{-2}	1.2959×10^{-6}	0.1340	18.580
$1.000 \times 10^{+0}$	1.2986×10^{-6}	0.1343	18.602
$1.000 \times 10^{+1}$	1.2976×10^{-6}	0.1342	18.604
$1.000 \times 10^{+2}$	1.2966×10^{-6}	0.1341	18.542

* standard finite element formulation

† corresponds to $P = 1/\kappa$

‡ corresponds to $P = 1/E$

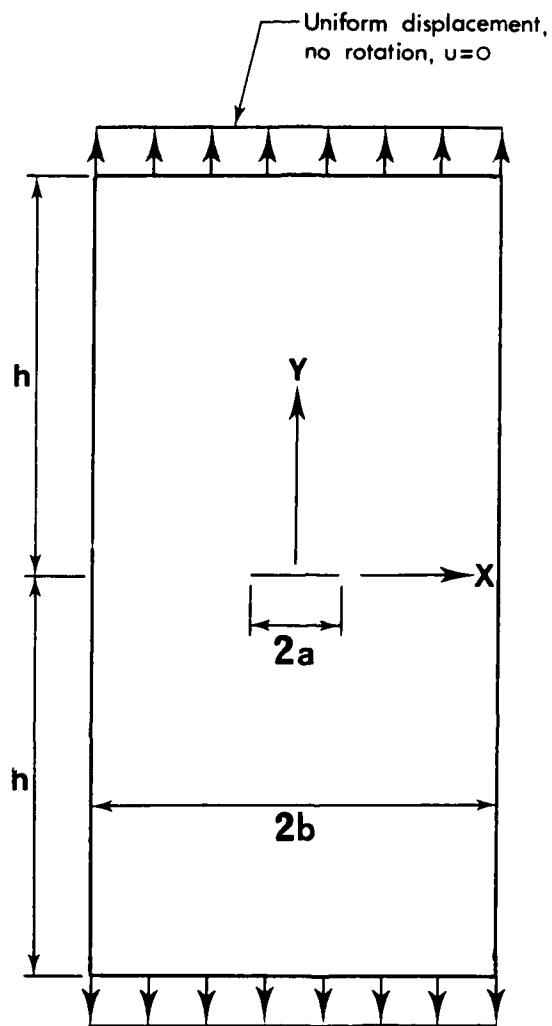


$a = 1.6667 \text{ mm}$

$b = 10.0 \text{ mm}$

$h = 10.0 \text{ mm}$

Fig. 1: Edge-cracked panel showing main dimensions.



$$a = 1.6667 \text{ mm}$$

$$b = 10.0 \text{ mm}$$

$$h = 10.0 \text{ mm}$$

Fig. 2: Centre-cracked panel showing main dimensions.

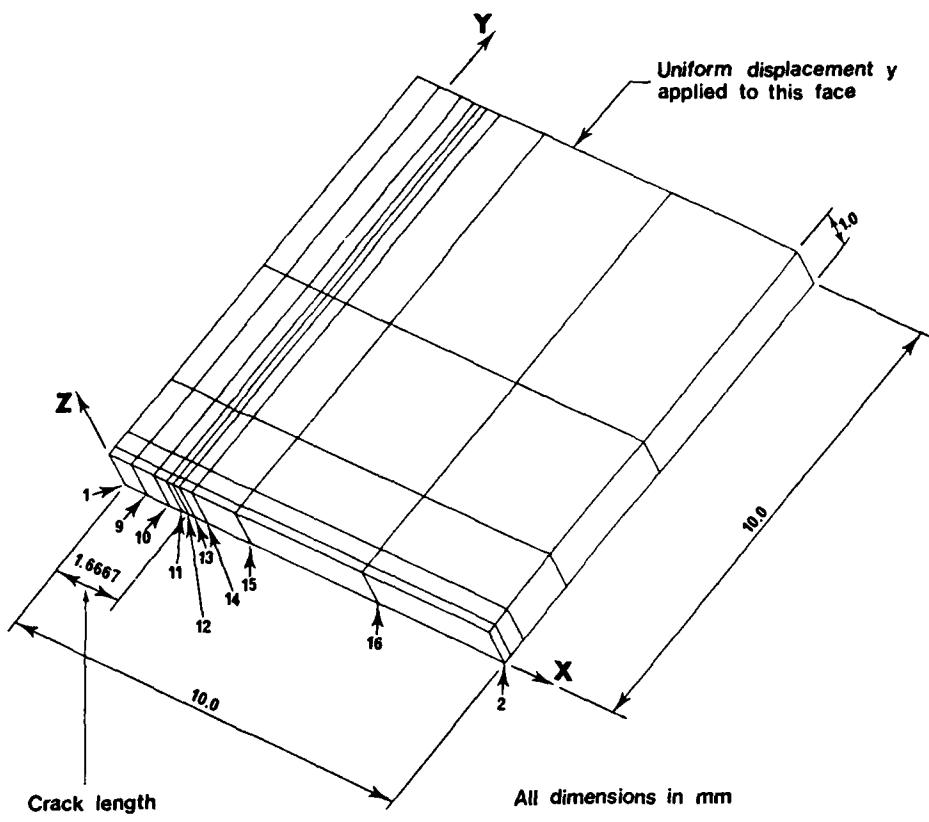


Fig. 3: Finite element mesh used to model the edge-cracked and centre-cracked panels.

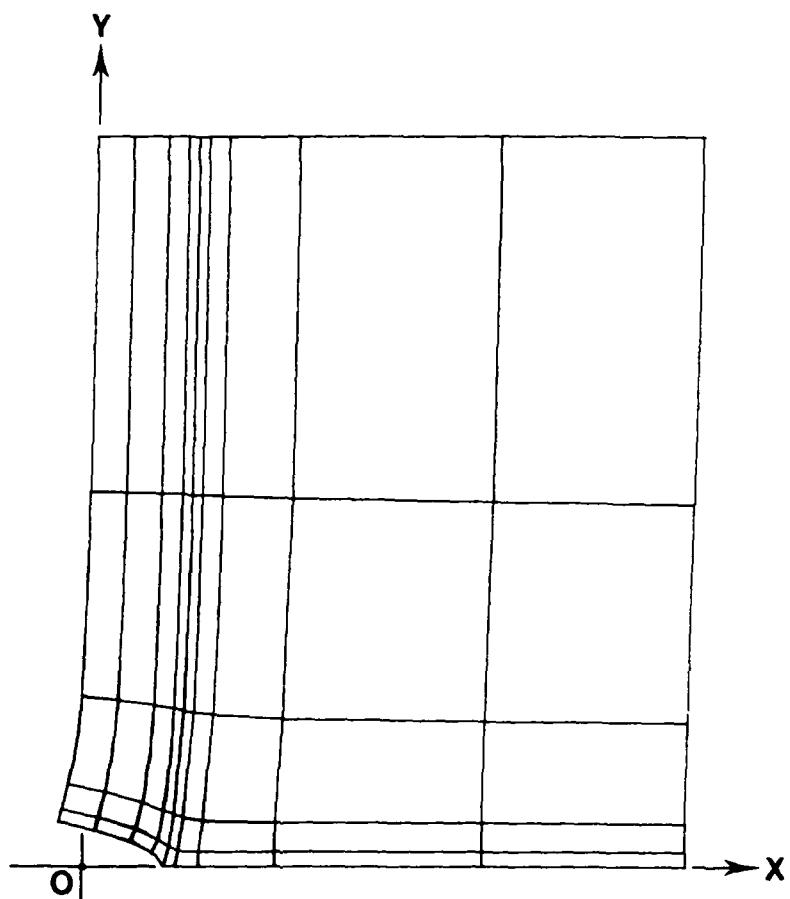


Fig. 4 : Typical deformed finite element mesh for the edge-cracked panel loaded by a uniform displacement.

DISTRIBUTION

AUSTRALIA

Department of Defence

Defence Central

Chief Defence Scientist
FAS Science Corporate Management (shared copy)
FAS Science Policy (shared copy)
Director, Departmental Publications
Counsellor, Defence Science, London (Doc Data Sheet Only)
Counsellor, Defence Science, Washington (Doc Data Sheet Only)
S.A. to Thailand MRD (Doc Data Sheet Only)
S.A. to the DRC (Kuala Lumpur) (Doc Data Sheet Only)
OIC TRS, Defence Central Library
Document Exchange Centre, DISB (18 copies)
Joint Intelligence Organisation
Librarian H Block, Victoria Barracks, Melbourne
Director General - Army Development (NSO) (4 copies)
Defence Industry and Materiel Policy, FAS

Aeronautical Research Laboratory

Director
Library
Chief - Aircraft Structures
Divisional File - Aircraft Structures
Authors: W. Waldman
T.E. Tay
R. Jones
B.C. Hoskin
A.A. Baker
J. Sparrow
A. Wong
L. Molent
J. Paul
T. Ryall
M. Heller
S. Dunn

Materials Research Laboratory

Director/Library

Defence Science & Technology Organisation - Salisbury
Library

Navy Office

Navy Scientific Adviser (3 copies Doc Data sheet only)

Army Office

Scientific Adviser - Army (Doc Data sheet only)

Air Force Office
Air Force Scientific Adviser
Engineering Division Library

Department of Transport & Communication
Library

Statutory and State Authorities and Industry

Aero-Space Technologies Australia,
Manager/Librarian (2 copies)
S. Georgiadis
Hawker de Havilland Aust Pty Ltd, Victoria, Library
Hawker de Havilland Aust Pty Ltd, Bankstown, Library
Rolls Royce of Australia Pty Ltd, Mr C.G.A. Bailey
Gas & Fuel Corporation of Vic., Manager Scientific Services
SEC of Vic., Herman Research Laboratory, Library
BHP, Melbourne Research Laboratories
Civil Aviation Authority
C. Torkington, Airworthiness Division, Canberra

Universities and Colleges

Adelaide
Barr Smith Library
Professor of Mechanical Engineering

Flinders
Library

LaTrobe
Library

Melbourne
Engineering Library

Monash
Hargrave Library
Prof I.J. Polmear, Materials Engineering

Newcastle
Library

New England
Library

Sydney
Engineering Library
Head, School of Civil Engineering

NSW
Physical Sciences Library
Professor R.A. Bryant, Mechanical Engineering
Library, Australian Defence Force Academy

Queensland
Library

Tasmania
Engineering Library

Western Australia
Library
Professor B.J. Stone, Head Mechanical Engineering

RMIT
Library

University College of the Northern Territory
Library

CANADA
CAARC Coordinator Structures

NRC
Aeronautical & Mechanical Engineering Library
Division of Mechanical Engineering Library

Universities and Colleges
Toronto
Institute for Aerospace Studies

FRANCE
ONERA, Library

GERMANY
Fachinformationszentrum: Energie, Physic, Mathematik GMBH

INDIA
CAARC Coordinator Materials
CAARC Coordinator Structures
Defence Ministry, Aero Development Establishment, Library
Hindustan Aeronautics Ltd, Library
National Aeronautical Laboratory, Information Centre

ISRAEL
Technion-Israel Institute of Technology
Professor J. Singer

ITALY
Professor Ing. Guiseppe Gabrielli

JAPAN
National Aerospace Laboratory
Institute of Space and Astronautical Science, Library

Universities
Kagawa University
Professor H. Ishikawa

NETHERLANDS
National Aerospace Laboratory (NLR), Library

NEW ZEALAND
Defence Scientific Establishment, Library

Universities
Canterbury
Library
Professor D. Stevenson, Mechanical Engineering

SWEDEN
Aeronautical Research Institute, Library
Swedish National Defense Research Institute (FOA)

SWITZERLAND
Armament Technology and Procurement Group
F+W (Swiss Federal Aircraft Factory)

UNITED KINGDOM
Ministry of Defence, Research, Materials and Collaboration

Royal Aircraft Establishment
Bedford, Library
Pyestock, Director
Farnborough, Dr G. Wood, Materials Department

National Physical Laboratory, Library
National Engineering Laboratory, Library
British Library, Document Supply Centre
CAARC Co-ordinator, Structures
Aircraft Research Association, Library
Fulmer Research Institute Ltd, Research Director
Rolls-Royce Ltd, Aero Division Bristol, Library

British Aerospace
Kingston-upon-Thames, Library
Hatfield-Chester Division, Library
British Hovercraft Corporation Ltd, Library
Short Brothers Ltd, Technical Library

Universities and Colleges
Bristol
Engineering Library

Cambridge
Library, Engineering Department
Whittle Library

London
Professor G.J. Hancock, Aero Engineering

Manchester
Professor, Applied Mathematics

Nottingham
Science Library

Southampton
Library

Strathclyde
Library

Cranfield Inst. of Technology
Library

Imperial College
Aeronautics Library

UNITED STATES OF AMERICA
NASA Scientific and Technical Information Facility
Materials Information, American Society for Metals

Boeing Company
Mr W.E. Binz
Mr J.C. McMillan
Kentex Research Library
United Technologies Corporation, Library
Lockheed-California Company
Lockheed Missiles and Space Company
Lockheed Georgia
McDonnell Aircraft Company, Library
Nondestructive Testing Information Analysis Center

Universities and Colleges
Chicago
John Crerar Library

Florida
Aero Engineering Department
Professor D.C. Drucker
Johns Hopkins
Professor S. Corrsin, Engineering

Iowa State
Dr G.K. Serovy, Mechanical Engineering

Iowa
Professor R.I. Stephens

Princeton
Professor G.L. Mellor, Mechanics

Massachusetts Inst. of Technology
MIT Libraries

SPARES (10 copies)
TOTAL (156 copies)

DOCUMENT CONTROL DATA

		PAGE CLASSIFICATION UNCLASSIFIED	
		PRIVACY MARKING	
1a. AR NUMBER AR-005-592	1b. ESTABLISHMENT NUMBER ARL-STRUC-R-438	2. DOCUMENT DATE FEBRUARY 1989	3. TASK NUMBER AIR 86/012
4. TITLE A PENALTY ELEMENT FORMULATION FOR CALCULATING BULK STRESS		5. SECURITY CLASSIFICATION (PLACE APPROPRIATE CLASSIFICATION IN BOX(S) IE. SECRET (S), CONF. (C) RESTRICTED (R), UNCLASSIFIED (U)).	6. NO. PAGES 18
		<input type="checkbox"/> U <input type="checkbox"/> U <input type="checkbox"/> U	7. NO. REFS. 3
8. AUTHOR(S) W. Waldman, T.E. Tay and R. Jones		9. DOWNGRADING/DELIMITING INSTRUCTIONS Not applicable.	
10. CORPORATE AUTHOR AND ADDRESS AERONAUTICAL RESEARCH LABORATORY P.O. BOX 4331, MELBOURNE VIC 3001		11. OFFICE/POSITION RESPONSIBLE FOR: SPONSOR _____ SECURITY _____ DOWNGRADING _____ APPROVAL _____ RAAF	
12. SECONDARY DISTRIBUTION (OF THIS DOCUMENT)		Approved for public release.	
OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH ASDIS, DEFENCE INFORMATION SERVICES BRANCH, DEPARTMENT OF DEFENCE, CANBERRA, ACT 2601			
13a. THIS DOCUMENT MAY BE ANNOUNCED IN CATALOGUES AND AWARENESS SERVICES AVAILABLE TO.... No limitations.			
13b. CITATION FOR OTHER PURPOSES (IE. CASUAL ANNOUNCEMENT) MAY BE		<input checked="" type="checkbox"/> X	UNRESTRICTED OR <input type="checkbox"/> AS FOR 13a.
14. DESCRIPTORS Thermoelasticity SPATE 8000 stress analyzer Finite element analysis		15. DRDA SUBJECT CATEGORIES 0046E	
16. ABSTRACT This report describes the formulation of a three-dimensional penalty element that uses the bulk stress at each node as a degree of freedom, in addition to the usual u, v, and w displacements. The penalty function method is used to define a modified energy functional that is constrained by the equation for bulk stress, and a solution is obtained by specifying stationarity of this modified functional. Edge-cracked and centre-cracked panels are modelled in order to investigate the practical implementation of the penalty			

PAGE CLASSIFICATION UNCLASSIFIED

PRIVACY MARKING

THIS PAGE IS TO BE USED TO RECORD INFORMATION WHICH IS REQUIRED BY THE ESTABLISHMENT FOR ITS OWN USE BUT WHICH WILL NOT BE ADDED TO THE DISTIS DATA UNLESS SPECIFICALLY REQUESTED.

16. ABSTRACT (CONT.)
element formulation and to compare the results with those obtained using standard methods.

17. IMPRINT

AERONAUTICAL RESEARCH LABORATORY, MELBOURNE

18. DOCUMENT SERIES AND NUMBER
AIRCRAFT STRUCTURES
REPORT 438

19. COST CODE
211080

20. TYPE OF REPORT AND PERIOD
COVERED

21. COMPUTER PROGRAMS USED

PAFEC finite element analysis package

22. ESTABLISHMENT FILE REF.(S)

23. ADDITIONAL INFORMATION (AS REQUIRED)